Equation (12) then can be solved easily to obtain, after imposing conditions (11)′,

$$f_1' = (C - \eta^2)/\cosh^2\eta \qquad (14)$$

where C is a free constant. This constant makes it possible to satisfy condition (11)″ exactly.

The value of C so determined, which represents the u_1 component of the velocity at $\eta = 0$, is $C = -0.428$ and agrees with the numerical result, $C = 0{,}425$, of Ref. 1. Moreover, comparison of the forementioned solution with that of Ref. 3 shows that the agreement holds for the whole field, the difference being only of some units in the third decimal place. It therefore follows that a second iteration is not necessary. The velocity function (14) is shown in Fig. 1 together with the Schlichting solution for the jet in a medium at test.[3]

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Torsional Vibration of a Semi-Infinite Viscoelastic Circular Cylinder Due to Transient Torsional Couple

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THIS paper is concerned with the determination of displacement in a semi-infinite viscoelastic cylinder when a torque, exponentially decreasing with time, is applied on a prescribed region of the plane end.

Method of Solution

Let b be the radius of the circular cylinder, and let the torque be applied at the plane boundary $Z = 0$. The material of this cylinder is supposed to satisfy the stress-strain relation

$$\tau_{ij} = [\lambda + \lambda'(\partial/\partial t)]e_{kk}\delta_{ij} + [\mu + \mu'(\partial/\partial t)]e_{ij} \qquad (1)$$

where τ_{ij} and e_{ij} are the stress and strain tensors, respectively, and

$$\delta_{ij} = 0 \qquad i \neq j$$
$$\delta_{ij} = 1 \qquad i = j$$

$\lambda, \lambda', \mu, \mu'$ being material constants.[1]

Choosing cylindrical coordinates with the Z axis along the axis of the cylinder, the components of displacement at any point of the cylinder are

$$u_r = 0 \qquad u_\theta = \vartheta(r,z,t) \qquad u_z = 0 \qquad (2)$$

Thus the components of strain are

$$e_{rr} = e_{\theta\theta} = e_{zz} = 0 \qquad e_{r\theta} = (\partial\vartheta/\partial r) - (\vartheta/r)$$
$$e_{\theta z} = \partial\vartheta/\partial z \qquad e_{rz} = 0 \qquad (3)$$

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From (1), the stress components are

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{zz} = 0 \qquad \tau_{r\theta} = [\mu + \mu'(\partial/\partial t)]e_{r\theta}$$
$$\tau_{\theta z} = [\mu + \mu'(\partial/\partial t)]e_{\theta z} \qquad \tau_{rz} = 0 \qquad (4)$$

The equations of motion take the form

$$\rho\frac{\partial^2\vartheta}{\partial t^2} = \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial\vartheta}{\partial r} - \frac{\vartheta}{r^2} + \frac{\partial^2\vartheta}{\partial z^2}\right) \qquad (5)$$

ρ being the density of the material, with the boundary conditions

$$\tau_{\theta z} = \left[\left(\mu + \mu'\frac{\partial}{\partial t}\right)\frac{\partial\vartheta}{\partial z}\right]_{z=0} = Pre^{-\Omega t}$$
$$t \geqslant 0, \Omega > 0 \text{ for } 0 \leqslant r \leqslant a$$
$$= 0 \qquad \text{for } a < r \leqslant b \text{ where } a < b \qquad (6)$$
$$\tau_{r\theta} = \left[\left(\mu + \mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial\vartheta}{\partial r} - \frac{\vartheta}{r}\right)\right]_{r=b} = 0 \qquad t \geqslant 0$$
$$\vartheta = 0 \qquad \text{at } z = \infty$$

Assuming

$$\vartheta = A_nJ_1(K_nr)\varphi_n(z,t) \qquad (7)$$

where K_nb is the nth root of $J_2(Kb) = 0$, Eq. (5) reduces to

$$\rho\frac{\partial^2\varphi_n}{\partial t^2} = \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left[\frac{\partial^2\varphi_n}{\partial z^2} - K_n^2\varphi_n(z,t)\right] \qquad (8)$$

To solve (8), take

$$\varphi_n(z,t) = f(z)e^{-\Omega t} \qquad (9)$$

From (8) and (9), one obtains

$$(d^2f/dz^2) - m^2f = 0 \qquad (10)$$

where $m^2 = (\rho\Omega^2 - K_n^2\Omega\mu' + K_n^2\mu)/(\mu - \mu'\Omega)$. It is assumed that $\mu > \mu'\Omega$.

Thus $f(z) = C_1e^{mz} + C_2e^{-mz}$. The last condition of (6) gives $C_1 = 0$, and $f(z)$ becomes

$$f(z) = C_2e^{-mz} \qquad (11)$$

Thus one writes

$$\vartheta = \sum_{n=1}^{\infty} B_n\, J_1(K_nr)e^{-mz}e^{-\Omega t} \qquad (12)$$

From the first condition of (6) and from (12), one obtains

$$\frac{Pr}{\mu - \mu'\Omega} = \sum_{n=1}^{\infty} C_nJ_1(K_nr) \qquad (13)$$

where $C_n = -B_nm$. C_n is given by the relation[2]

$$C_n\{(K_n^2a^2 - 1)J_1^2(K_na) + a^2K_n^2J_1'^2(K_na)\} = 2K_n^2\int_0^a \frac{Pr^2}{\mu - \mu'\Omega}J_1(K_nr)dr = \frac{2K_n^2P}{(\mu - \mu'\Omega)}\cdot\frac{a^2}{K_n}J_2(K_na)$$

Therefore

$$C_n = \frac{2K_nPa^2J_2(K_na)}{(\mu - \mu'\Omega)[(K_n^2a^2 - 1)J_1^2(K_na) + a^2K_n^2J_1'^2(K_na)]}$$

From (13)

$$B_n = -C_n/m$$
$$= \frac{-2k_nPb^2J_2(K_na)}{(\mu - \mu'\Omega)m[(K_n^2a^2 - 1)J_1^2(K_na) + a^2K_n^2J_1'^2(K_na)]} \qquad (14)$$

From relations (12) and (14), the displacement is of the form

$$\vartheta = \sum_{n=1}^{\infty} \frac{-2K_nPa^2J_2(K_na)J_1(K_nr)e^{-mz}e^{-\Omega t}}{(\mu - \mu'\Omega)m\{(K_n{}^2a^2 - 1)J_1{}^2(K_na) + a^2K_n{}^2J_1{}'^2(K_na)\}}$$
$$= \frac{-2Pa^2e^{-\Omega t}}{(\mu - \mu'\Omega)} \sum_{n=1}^{\infty} \frac{K_nJ_2(K_na)J_1(K_nr)e^{-mz}}{m[(K_n{}^2a^2 - 1)J_1{}^2(K_na) + a^2K_n{}^2J_1{}'^2(K_na)]} \quad (15)$$

From (15) and (4), one obtains

$$\tau_{\theta z} = [\mu + \mu'(\partial/\partial t)](\partial\vartheta/\partial z)$$
$$= 2e^{-\Omega t}\,\rho a^2 \cdot \sum_{n=1}^{\infty} \frac{K_nJ_2(K_na)J_1(K_nr)e^{-mz}}{\{(K_n{}^2a^2 - 1)J_1{}^2(k_na) + a^2K_n{}^2J_1{}'^2(K_na)\}} \quad (16)$$

$$\tau_{\theta r} = \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial\vartheta}{\partial r} - \frac{\vartheta}{r}\right) =$$
$$-2pa^2e^{-\Omega t}\left\{\sum_{n=1}^{\infty} \frac{K_n{}^2J_2(K_na)J_0(K_nr)e^{-mz}}{m[(K_n{}^2a^2 - 1)J_1{}^2(K_na) + a^2K_n{}^2J_1{}'^2(K_na)]} - \frac{2}{r}\sum_{n=1}^{\infty} \frac{K_nJ_2(K_na)J_1(K_nr)}{m[(K_n{}^2a^2 - 1)J_1{}^2(K_na) + a^2K_n{}^2J_1{}'^2(K_na)]}\right\} \quad (17)$$

On the plane boundary $Z = 0$, the displacement, from (15), is

$$[\vartheta]_{z=0} = \frac{-2pa^2e^{-\Omega t}}{(\mu - \mu'\Omega)} \sum_{n=1}^{\infty} \frac{K_nJ_2(K_na)J_1(K_nr)}{m[(K_n{}^2a^2 - 1)J_1{}^2(K_na) + a^2K_n{}^2J_1{}'^2(K_na)]} \quad (18)$$

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Method of Evaluating Script *F* for Radiant Exchange within an Enclosure

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A method of evaluating script *F* for radiant exchange within an enclosure by a single matrix inversion is developed. The method lends itself well to problems handled analytically or to problems solved by the use of a digital computer.

Nomenclature

a_{jk} = element of the inverse matrix $[\beta]^{-1}$ (jth row, kth column), dimensionless
A_j = area of the jth surface, ft²
F_{jk} = shape factor from jth surface to kth surface
J_j = radiosity of the jth surface
$q_{j,k}$ = net thermal radiation exchange between the jth surface and the kth surface, Btu/hr
ϵ_j = emissivity of the jth surface
ρ_j = reflectivity of the jth surface
σ = Stefan-Boltzmann constant

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THE thermal interchange between diffuse surfaces in an enclosure usually is evaluated by one of the two methods, the network or the script F method. Both methods are established well and are presented in Refs. 1 and 2. It is shown readily that the basic equations of the net radiation concept as formulated by Poljak[3] leads either to the Oppenheim network or to the script F concept employed by Hottel.

Coupling of the radiation network which is described by n-simultaneous linear equations (linear in terms of σT^4 and n is the number of surfaces) with the temperature-time dependent equations of conduction and convection is somewhat unwieldy since, at each interval of time, the n-simultaneous equations must be solved. It is simpler to employ the concept of script F and linearize the thermal radiation exchange.

Analysis

The basis of the script F method is that the net exchange between surfaces A_j and A_k in an enclosure must be of the form $\sigma(T_j{}^4 - T_k{}^4)$ multiplied by some factor $\mathfrak{F}_{jk}$ (called script F) which depends upon the geometry and the emissivity of the surfaces. Thus the net exchange between two surfaces identified by the subscripts j and k can be expressed in the form

$$q_{j,k} = A_j\mathfrak{F}_{jk}\,(\sigma T_j{}^4 - \sigma T_k{}^4) \quad (1)$$

Script F, $\mathfrak{F}_{jk}$, is found from the n-simultaneous equations that describe the Oppenheim network. It is shown readily that the set of radiosity equations are

$$J_j - \epsilon_j\,\sigma T_j{}^4 = \rho_j \sum_{k=1}^{n} F_{jk}\,J_k \qquad j = 1, 2, 3, \ldots n \quad (2)$$

If all the terminals ($\sigma T_k{}^4$ is a potential at the terminal k) are grounded except the jth, which is fixed at unity potential, the net flow, $q_{j,k}$, into the grounded kth terminal is

$$(J_k\,\epsilon_k\,A_k)/\rho_k \quad (3)$$

This relationship can be verified easily by examining the network.

Since $\sigma T_j{}^4 = 1.0$ and $\sigma T_k{}^4 = 0$, $k = 1, 2, \ldots 3, \ldots n$, $k \neq i$, it is seen from (1) and (3) that

$$q_{j,k}' = A_j \qquad \mathfrak{F}_{jk} = J_k\,\epsilon_k\,A_k/\rho_k$$

or

$$\mathfrak{F}_{jk} = J_k\,\epsilon_k\,A_k/\rho_k\,A_j \quad (4)$$

Radiosity, J_k, is found from the set of Eqs. (2) with the terminals grounded except the jth, which is set at unity potential. In matrix form, this can be expressed as

$$[\beta]\begin{Bmatrix} J_1 \\ \cdot \\ \cdot \\ \cdot \\ J_k \\ \cdot \\ \cdot \\ \cdot \\ J_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_j \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} \quad (5)$$

or

$$\begin{Bmatrix} J_1 \\ J_k \\ J_n \end{Bmatrix} = [\beta]^{-1}\begin{Bmatrix} 0 \\ \epsilon_j \\ 0 \end{Bmatrix} \quad (6)$$